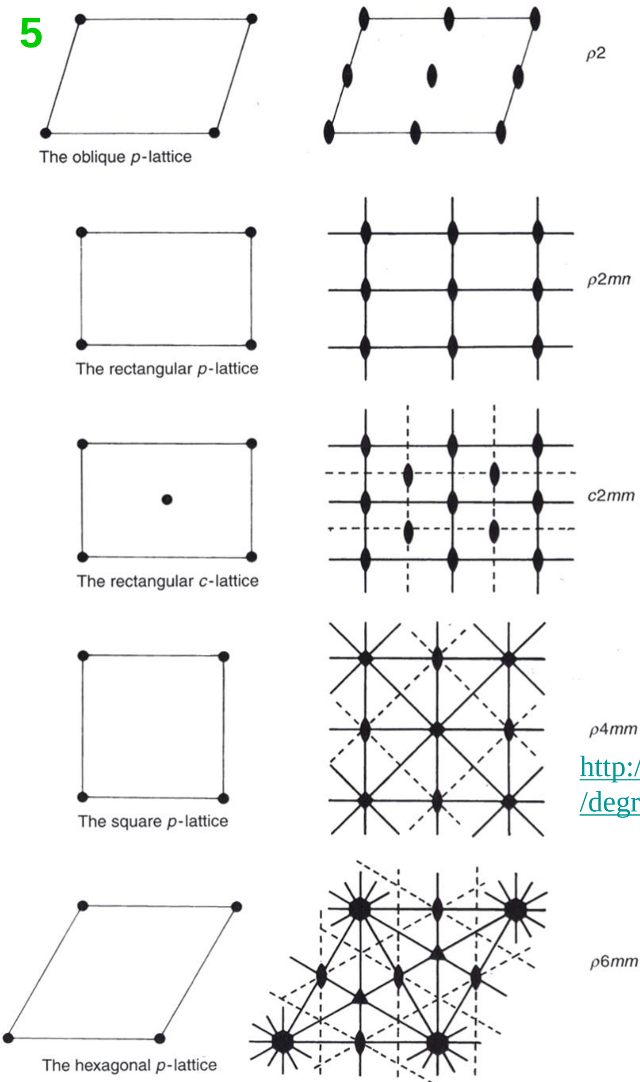
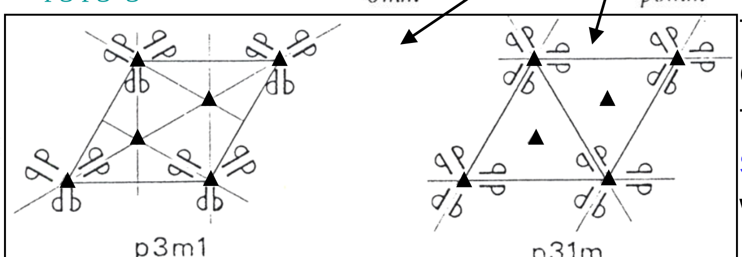
The 17 *2-D* Plane (Space) Groups based on rotation axes and mirror/glide planes

•The combination of a point group with the symmetry of a lattice (previously discussed 5 *2-D* ones) gives rise to a space group, or in *2-D* a plane group → there are 17 *2-D* plane groups:



System (Lattice type) 5	Point group 10	17 Plane group: 13 Based directly on point groups	+ 4 Glide-reflection groups	
Oblique (p Parallelogram)	1 2	p1 p2		
Rectangular (p Rectangular) (c Rectangular)	m	pm cm	pg	
	2mm	p2mm c2mm	p2mg p2gg	Sometimes abbreviated by omitting the '2'.
Square (p Square)	4	p4		
	4mm	p4mm	p4gm	
Hexagonal (p Triequiangular)	3	p3		
	3m	p3m1 p31m		
	6 6mm	p6 p6mm		

http://neon.mems.cmu.edu/degraef/pg/pg_gif.html



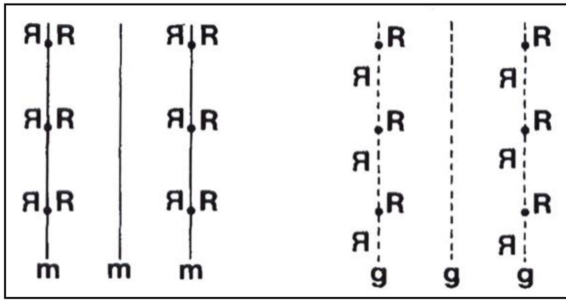
These 2 groups arise because the point group mirror planes can be aligned parallel to either the medians (p3m1) of **or** the sides (p31m) of the triangle which make up the triangular lattice.

The 17 *2-D* Plane (Space) Groups based on rotation axes and mirror/glide planes (cont.)

•*It is possible for different symmetry elements to interact to produce new elements*, for example in plane groups it is possible for a mirror plane to combine with translational symmetry of the lattice to produce what is called a glide(translate)-reflection element.

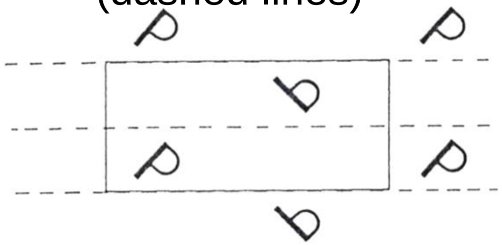
•Consider two patterns on right, which both have a *rectangular lattice*:

•The mirror (**m**) plane (*solid line*) is one of the 10 point groups, while a translation of a half-lattice spacing results in a glide (**g**) line of symmetry (*dashed line*).

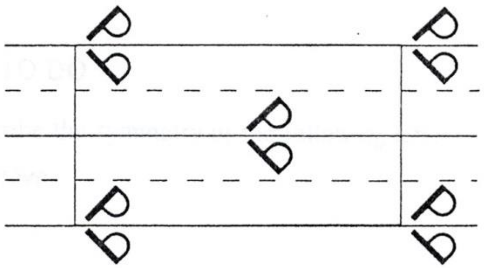


pm *pg*

Plane group *pg*:
(dashed lines)

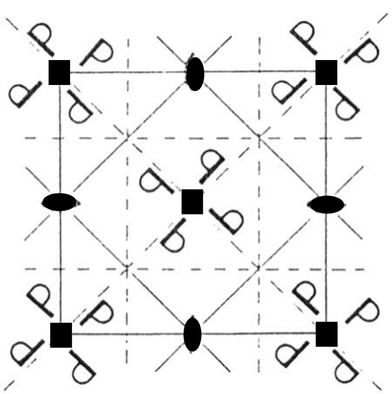


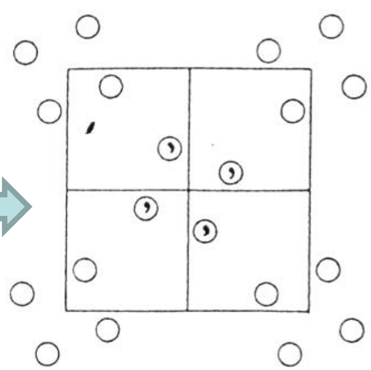
Plane group *cm*:



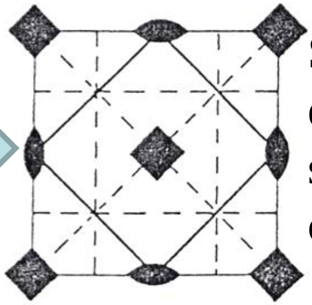
"Contains a set of glide planes which do not appear in the designation, since mirror planes take precedence over glide planes parallel to them."

Plane group *p4gm*:





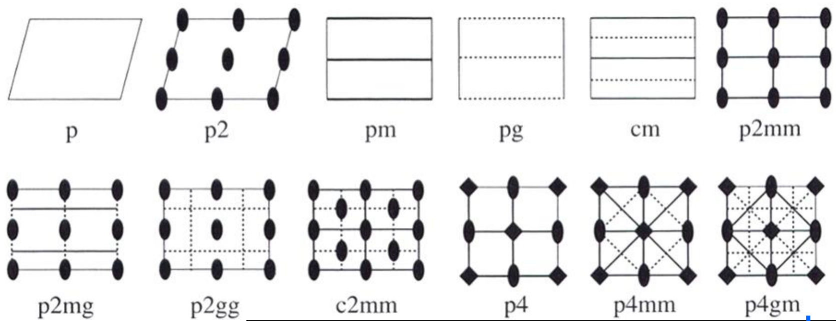
Use *"o" for element, e.g. P, (or RH)* and "," for mirrors *(or LH)*

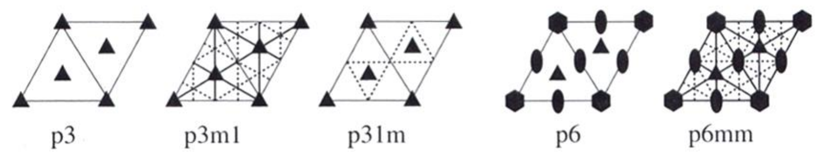


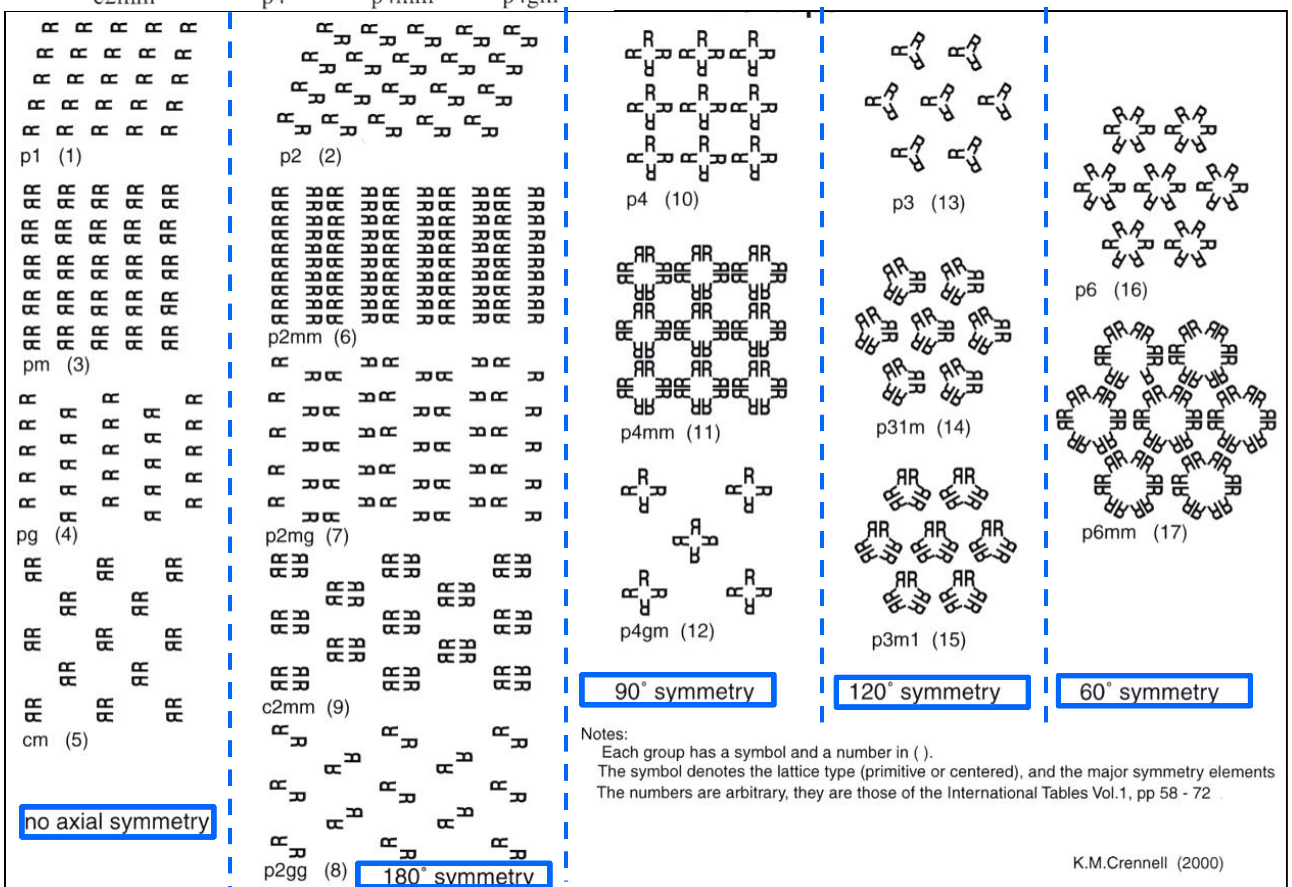
Symmetry elements in separate unit cell

4.9 The plane group *p4gm* as illustrated in *International Tables of X-ray Crystallography.*

The 17 *2-D* Plane (Space) Groups based on rotation axes and mirror/glide planes (cont.)







Summary of the 17 *2-D* Plane (Space) Groups based on rotation axes and mirror/glide planes

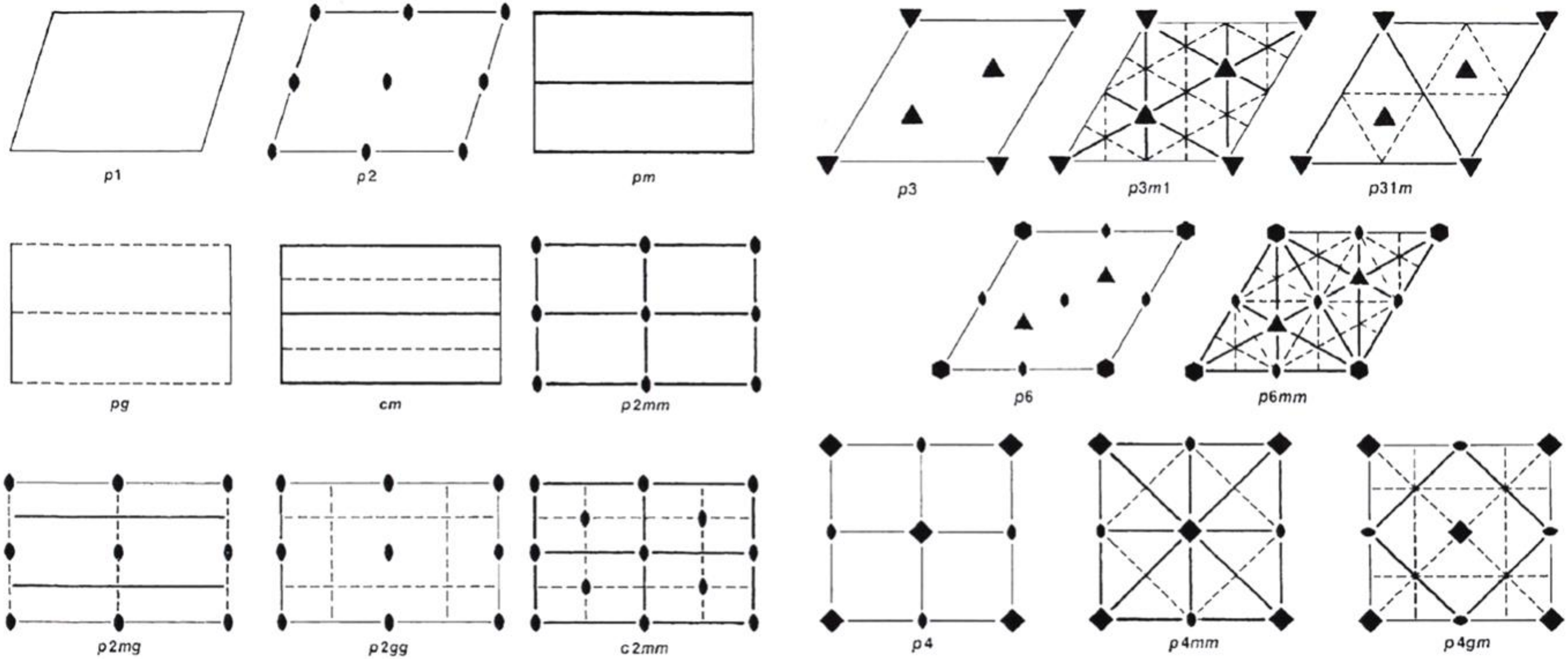


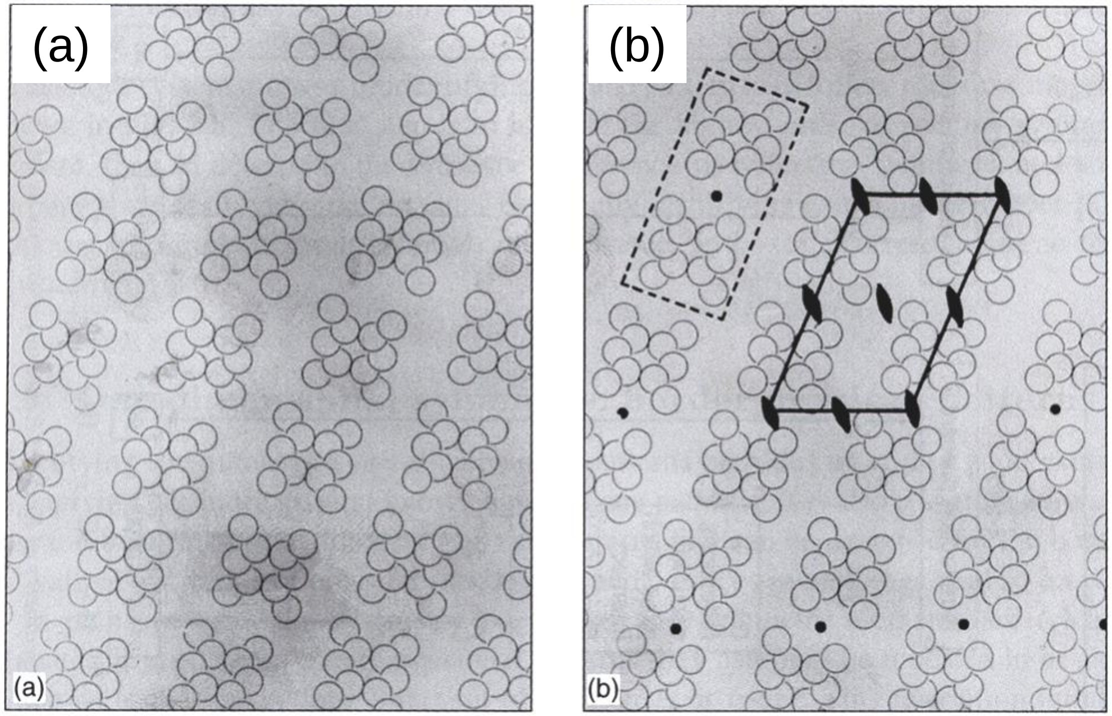
Fig. 2.6. (a) The seventeen plane groups (from *Point and Plane Groups* by K. M. Crennell). The numbering 1–17 is that which is arbitrarily assigned in the International Tables. Note that the 'shorthand' symbols do not necessarily indicate all the symmetry elements which are present in the patterns. (b) The symmetry elements outlined within (conventional) unit cells of the seventeen plane groups, heavy solid lines and dashed lines represent mirror and glide lines respectively (from *Manual of Mineralogy* 21st edn, by C. Klein and C. S. Hurlbut, Jr., John Wiley, 1999).

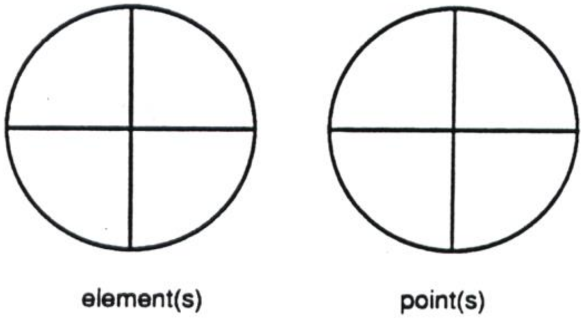
Can superimpose these symmetry elements on top of slide 3 symbols ("R"), which are the equipoints = equivalent points

Recognizing motifs (basis), symmetry elements and lattice types in 2-D

•This is essential to find which of the 17 plane groups they belong.

•Any regular, patterned object will suffice. For example in (a) and (b):





•<u>You should recognize that the molecules, or groups of atoms, aren't identical in these 2-D projections</u>.

•The motif is a pair of such molecules and this is the "unit of pattern" that is repeated (i.e. it's the **<u>best</u>** unit cell→ *represents the highest symmetry reflected of the pattern/image/motif*).

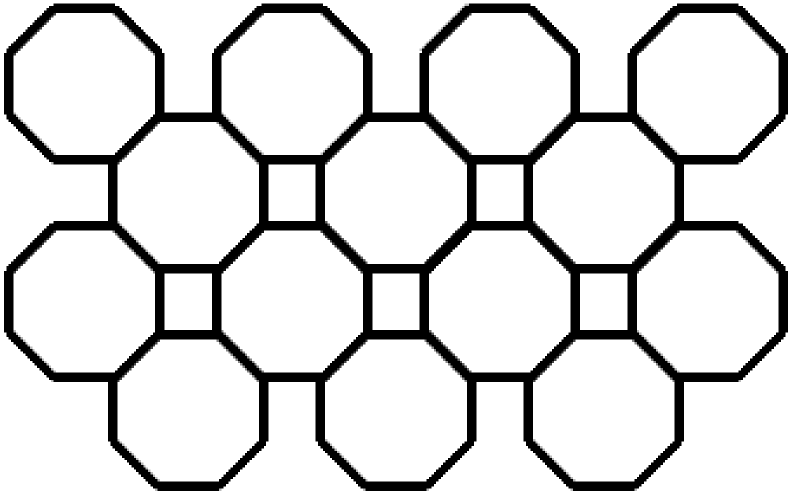
•Now look for symmetry elements and indicate the positions of all of these on the pattern in (b).

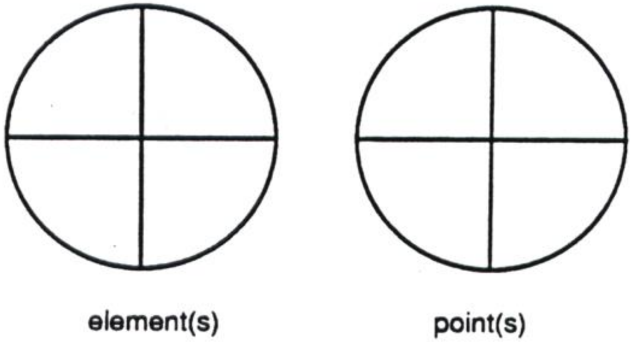
•Finally, insert the lattice points – one for each motif. Anywhere will do, but it is convenient to have them coincide with a symmetry element.

•2-D crystal system=<u>oblique</u>;2-D Bravais lattice=<u>*p*-oblique</u>;2-D point group=<u>2</u>, 2-D plane group=<u>*p*2</u>

Another Example of recognizing motifs (basis), symmetry elements and lattice types in 2-D

•This is essential to find which of the 17 plane groups they belong (the end result).
•Any regular, patterned object will suffice. For example:





•The motif is the "unit of pattern" that is repeated (i.e. it's the **best** unit cell→ *represents the highest symmetry reflected of the pattern/image/motif*).
•Now look for symmetry elements and indicate the positions of all of these on the pattern.
•Finally, insert the lattice points – one for each motif. Anywhere will do, but it is convenient to have them coincide with a symmetry element.
•Draw at least 6 lattice points and show the best unit cell. Describe the cells and their centering, if any.
•Inspect the pattern above and locate any rotational and/or mirror symmetry (show some of it on the drawings above).
•The 2-D crystal system=?; 2-D Bravais lattice=?, 2-D point group=?, 2-D plane group=?